SCIENTIFIC OASIS

Decision Making: Applications in
Management and Engineering

ISSN: 2560-6018, eISSN: 2620-0104

DECISION MAKING: APPLICATIONS IN MANAGEMENT AND ENGINEERING

EVALUATION OF PERFORMANCE MEASURES OF A BULK ARRIVAL PRIORITY QUEUEING MULTI-SERVER MODEL USING α-CUT METHOD

G. SOMASUNDARA ORI [1] and B.ABIRAMI *[2]

1,2 Dwaraka Doss Goverdhan Doss Vaishnav College,

Affiliated To University Of Madras, Chennai, Tamilnadu, India.

1)soamsundaraori@gmail.com&2) abirami.balagurunathan@gmail.com

***2 corresponding author**

ABSTRACT:

In this study, the parametric programming problem approach has been used to determine the membership function of the system performance measure with bulk arrivalpriority queue. Based on Zadeh's extension principle, this model can be used to transform a family of bulk arrival crisp priority queues from a bulk arrival priority fuzzy queue. Using an α-cut strategy with trapezoidal and triangular fuzzy numbers, a parametric programming problem has been applied to solve the fuzzy bulk arrival priority queue using the $FM^b/FM/S$ model. The model is verified by working through a numerical example.

Key words: Bulk Queuing model, priority queues, parametric programming problem, triangular fuzzy number, trapezoidal fuzzy number.

AMS Mathematics Subject Classification (2010): 90B22; 60K25; 68M20; 03E72

1. Introduction

Queuing theory is the mathematical study of how waiting lines or queues form. Certain consumers are given preference for the service in a priority queue over others, irrespective of their arrival time. It is very hard to evaluate the efficiency of such a queuing system by utilizing traditional natural processes when the arrival rate and service rate are not numerical in real applications. Fuzziness was suggested by Zadeh[14] as a solution for these kinds of queues.

Earlier research on fuzzy queuing models mostly focused on simple queues with one or two fuzzy variables. The queuing model with priority has more uses when fuzzy parameters are added. In the fuzzy priority queuing system, Type I and Type II customers come with the Poisson process at rates of $\widetilde{\lambda_1}$ and $\widetilde{\lambda_2}$, respectively. The service time is distributed with $\tilde{\mu}$ under exponential distribution. Customers classed as Type I have the highest priority, while customers labelled as less priority are ranked lower.

In most queuing systems, customers arrive to the system one at a time (single-unit arrival). However, in real-world scenarios, service requests come in groups or bulk or batches. If these queuing models could be used for fuzzy batch-arrival queues rather than the more popular crisp batch-arrival queues that use multiple servers, we might find greater applications. In contrast to much previous research on fuzzy queuing models, which concentrated on simple queues, in this work we took into account the multi-server model with bulk fuzzy queues and priority service for customers. Fuzzy batch priority queues are another name for the fuzzy bulk queues.

Researchers like Chanas[1], Kao et al[7], Kaufmann [8,], R.J. Li, E.S. Lee[9], Negi and Lee [12] have all studied about fuzzy queuing models. M.L. Chaudhary [2], A. Nagoor Gani, V. Ashok Kumar [10], R. Sharma, and G.C. Sharma [13] all discussed the fuzzy bulk queues. Many factors related to fuzzy priority discipline queuing system has been provided by J.Devaraj, D.Jayalakshmi[3], B.Kalpana, N.Anusheela [5], B. Kalpana [6]. Jau-Chuan Ke, Hsin-I Huang, Chuen-Horng Lin [4], Narayanamoorthy and Ramya [11].J. Devaraj, D. Jayalakshmi, B. Kalpana, N. Anusheela, and B. Kalpana used triangular fuzzy numbers in the queuing model with the aid of the α-cut approach. The fuzzy queues were transformed into a family of crisp queues using α-cut and Zadeh's extension principle. The family of crisp queues is derived for different values of α and are solved using parametric programming problem.

This paper applies fuzzy bulk-queuing theory to construct the membership function of fuzzy priority queues. Utilizing the α-cut technique, fuzzy arithmetic operations, and triangular and trapezoidal fuzzy numbers, we offer a fuzzy bulk arrival size 'b' that is serviced by multiple servers with type 1 high priority and type 2 less priority or no priority. By transforming the conventional crisp batch-arrival priority queues into fuzzy batch-arrival priority queues with multiple servers, these queuing models can be applied to a variety of applications.

The structure of the paper is as follows. We recall the basic definition and notion of the fuzzy queuing system in Section 2. In Section 3, we provide a mathematical model for priority queues with multiple servers as well as a formulation for the fuzzy batch priority queues. Section 4 illustrates a model with a numerical example, and Section 5 proposes a conclusion.

2. Preliminaries

In this section we recall some basic definition.

Definition 2.1. Fuzzy set

Let X be a nonempty set. A fuzzy set$\tilde{A}$ in X is characterized by its membership function $\mu_{\tilde{A}}: X \to [0,1]$ and $\mu_{\tilde{A}}(x)$ is interpreted as the degree of membership of element x in fuzzy set $\tilde{A}$ for each x ∈ X. It is clear that A is completely determined by the set of tuples $\tilde{A}$={(x,$\mu_{\tilde{A}}(x)$)/ x∈X}.

Definition 2.2 α-cut

The α-cut for a fuzzy set $\tilde{A}$ are shown by $\tilde{A}_\alpha$ for $\alpha \in [0,1]$ are defined to be $\tilde{A}_\alpha = \{x/\mu_{\tilde{A}}(x) \geq \alpha, x \in X\}$, where X is the universal set. Upper and Lower bounds for any α-cut $\tilde{A}_\alpha$ are shown by $sup\ \tilde{A}_\alpha$ and $inf\ \tilde{A}_\alpha$ respectively.

Definition2.3.The crisp set

The crisp set of elements that belong to the fuzzy set $\tilde{A}$ at least to the degree α is called the α level set $\tilde{A}_\alpha = \{x \in X: \mu_{\tilde{A}}(x) \geq \alpha\}$ where $\alpha \in [0,1]$.

Definition2.4. A fuzzy number

A fuzzy set $\tilde{A}$ is defined on R, the set of real numbers is called a fuzzy number if its membership function $\mu_{\tilde{A}}: r \to [0,1]$ has the following conditions:

(a) $\tilde{A}$ is convex, which means that there exist $x_1, x_2 \in R$ and $\lambda \in [0,1]$
Such that $\mu_\lambda\{\lambda x_1 + (1-\lambda)x_2\} \geq \min\{\mu_\lambda(x_1), \mu_\lambda(x_2)\}$

(b) $\tilde{A}$ is normal, which means that there exist an $x \in R$ such that $\mu_\lambda(x) = \tilde{1}$.

$(c)\tilde{A}$ is piecewise continuous.

Definition2.5. Triangular fuzzy number

A fuzzy number $\tilde{A} = (x_1, x_2, x_3)$ is called triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < x_1 \\ \dfrac{x - x_1}{x_2 - x_1}, & x_1 \leq x \leq x_2 \\ \dfrac{x_3 - x}{x_3 - x_2}, & x_2 \leq x \leq x_3 \\ 0, & x > x_3. \end{cases}$$

Definition2.6. Conversion of triangular fuzzy number into Interval using $\alpha -$ cut

Let $\tilde{A} = (x_1, x_2, x_3)$ be the triangular fuzzy number then to fine α $-$cut of $\tilde{A}$. we first set α equal to the left and right membership function of $\tilde{A}$.

That is $\alpha = \frac{x-x_1}{x_2-x_1}$ and $\alpha = \frac{x_3-x}{x_3-x_2}$. Expressing x in terms of α we have,

$x = \alpha(x_2 - x_1) + x_1$ and $x = -\alpha(x_3 - x_2) + x_3$.

Therefore we can write the fuzzy interval in terms of $\alpha -$ cut interval:

$\tilde{A}_\alpha = [\alpha(x_2 - x_1) + x_1 \ , \ -\alpha(x_3 - x_2) + x_3]$.

Definition2.7. Trapezoidal fuzzy number

Trapezoidal fuzzy number can be defined as $\tilde{A} = (x_1, x_2, x_3, x_4)$ the membership function of this fuzzy number will be interpreted as follows.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < x_1 \\ \dfrac{x - x_1}{x_2 - x_1}, & x_1 \le x \le x_2 \\ 1 & x_2 \le x \le x_3 \\ \dfrac{x_4 - x}{x_4 - x_3}, & x_3 \le x \le x_4 \\ 0, & x > x_4 \end{cases}$$

Definition2.8. Conversion of Trapezoidal fuzzy number into Interval using $\alpha -$ cut

Let $\tilde{A} = (x_1, x_2, x_3, x_4)$ be the Trapezoidal fuzzy number then to fine α $-$cut of $\tilde{A}$. we first set α equal to the left and right membership function of $\tilde{A}$.

That is $\alpha = \frac{x-x_1}{x_2-x_1}$ and $\alpha = \frac{x_4-x}{x_4-x_3}$. Expressing x in terms of α we have,

$x = \alpha(x_2 - x_1) + x_1$ and $x = -\alpha(x_4 - x_3) + x_4$.

Therefore we can write the fuzzy interval in terms of $\alpha -$ cut interval:

$\tilde{A}_\alpha = [\alpha(x_2 - x_1) + x_1 \ , \ -\alpha(x_4 - x_3) + x_4]$.

Definition2.9. (FM/FM/S): (FCFS/∞/∞) Model

Multi server fuzzy queue infinite calling source and first come first served discipline. In technically (FM/FM/S): (FCFS/∞/∞). Multi server queue has two or more service facility in parallel providing identical service. All the customers in the waiting line will be served by multi server. The arrival time and the service time follow poison and exponential distribution. The performance measures of the Multi server queuing system are:

(a) Expected number of customers waiting in the queue

$$L_q = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^s}{(s-1)!\,(s\mu-\lambda)^2}P_0$$

(b) Average waiting time of a customer in the queue

$$W_q = \frac{\mu\left(\frac{\lambda}{\mu}\right)^s}{(s-1)!\,(s\mu-\lambda)^2}P_0$$

Here $$P_0 = \left[\sum_{n=0}^{s-1}\frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!}\frac{s\mu}{(s\mu-\lambda)}\right]^{-1}$$

3. Mathematical model for fuzzy batch priority queues with multi server

We consider fuzzy batch priority queueing model$FM^b/FM/$S. Every arrival is designated as a member of any one of two priority types. Then, we assumed that, in batches of size "b", the arrivals of the type 1 or higher priority have mean arrival rate ($\widetilde{\lambda_1}$) and the type 2 less priorityor no priority have mean arrival rate ($\widetilde{\lambda_2}$) such that $\tilde{\lambda} = \widetilde{\lambda_1} + \widetilde{\lambda_2}$.

The type 1 customer has the right to receive service prior to all other customers. We considered that the system's capacity and the calling source for the population is infinite, and that a multi-server system with an 'S' service channel would be used when customers arrived in batches of a finite size 'b'. Both the system capacity and the population of calling sources are infinite in the priority queues using fuzzy priority queues methods investigated by Devaraj & Jayalakshmi [10].

According to our model, the system's bulk arrival rate is finite, and each batch has a size of 'b'. At a multi– server facility in batches as a Poisson process with batch arrival mean rate$\tilde{\lambda}$, where $\tilde{\lambda}$,, is a fuzzy number and all service times are identically distributed according to an exponential distribution with fuzzy service rate with $\tilde{\mu}$. Customers are served according to two types of a priority model and the size of calling population is infinite and the system capacity is finite. Using α- cuts, the triangular arrival rate can be represented by different levels of interval of confidences.

3.1 Fuzzy priority batch queue with multi server

In $FM^b/FM/S$ a multi-server queuing system with priority, the inter arrival rate $\widetilde{AR}_i, \mathrm{i} = 1,2$ of units in the type 1 and type 2 priority and service rate $\widetilde{SR}$are represented by the fuzzy sets in the following form

$\widetilde{AR}_i = \{(a, \mu_{\widetilde{AR}_i}(a))/a \in M\}, i = 1,2$……………………………………(1)

$\widetilde{SR} = \{(s, \mu_{\widetilde{SR}}(s))/s \in N\}$……………………………………(2)

Where M is universal crisp set of the inter arrival rate with its membership function $\mu_{\widetilde{AR}_i}(a), i = 1,2$and N is also crisp universal set of service time with its membership function $\mu_{\widetilde{SR}}(s)$.

3.2 α − cuts of Fuzzy priority batch queue arrival rate and service rate

The α − cuts of arrival rate $\widetilde{AR}_i, \mathrm{i} = 1,2$and service rate $\widetilde{SR}$are denoted by

$\widetilde{AR}_i(\alpha) = \{a \in M/\mu_{\widetilde{AR}_i}(a) \geq \alpha\}, i = 1,2$**…………………………………..(3)**

$\widetilde{SR}(\alpha) = \{s \in N/\mu_{\widetilde{SR}}(s) \geq \alpha\}$**…………………………………..(4)**

Where $0 < \alpha \leq 1$. The $AR_i(\alpha), \mathrm{i} = 1,2$ and $\mathrm{S}R(\alpha)$ are the crisp sets and they

can be represented through different levels of confidence intervals [0,1]

using α − cut. These two sets expressing the relationship between the crisp sets and fuzzy sets.

3.3 Confidence intervals of Fuzzy priority batch queue arrival rate and service rate

The confidence intervals of the fuzzy sets $\widetilde{\mathrm{AR}}_\mathrm{i}, \mathrm{i} = 1,2$ and $\widetilde{\mathrm{SR}}$ are given by

$[\mathrm{l}_{\mathrm{AR_i(\alpha)}}, \mathrm{u}_{\mathrm{AR_i(\alpha)}}], \mathrm{i} = 1,2$ and $[\mathrm{l}_{\mathrm{SR(\alpha)}}, \mathrm{u}_{\mathrm{SR(\alpha)}}]$ respectively. Here the $\widetilde{\mathrm{AR}}_\mathrm{i}$ and $\widetilde{\mathrm{SR}}$ are fuzzy numbers, by Zadeh's extension principle [14], the membership function of the performance measure $\mathrm{pm}(\widetilde{\mathrm{AR}}_\mathrm{i}, \widetilde{\mathrm{SR}}), \mathrm{i} = 1,2$ is defined as

$\mu_{\mathrm{pm}(\widetilde{\mathrm{AR}}_\mathrm{i},\widetilde{\mathrm{SR}})}(z) = \sup_{a \in M, s \in N} Min\{\mu_{\widetilde{AR}_i}(a), \mu_{\widetilde{SR}}(s)/z = pm(a, s)\}, \mathrm{i} = 1,2$**….(5)**

Construction of the membership function $\mu_{\mathrm{pm}(\tilde{\mathrm{A}},\tilde{\mathrm{S}})}(z), i = 1,2$ is equivalent to derivation of $\alpha -$ cuts of $\mu_{\mathrm{pm}(\widetilde{\mathrm{AR}}_\mathrm{i},\widetilde{\mathrm{SR}})}$. From the equation (5), the equation

$\mu_{\mathrm{pm}(\widetilde{\mathrm{AR}}_\mathrm{i},\widetilde{\mathrm{SR}})}(z) = \alpha, i = 1,2$ is true only when either $\mu_{\widetilde{AR}_i}(a) = \alpha, \mu_{\widetilde{SR}}(s) \geq \alpha$ or $\mu_{\widetilde{AR}_i}(a) \geq \alpha, \mu_{\widetilde{SR}}(s) = \alpha$ is true.

3.4 Construction of parametric programming problem for fuzzy numbers

After defining performance measure's membership function, Zadeh's extension principle and parametric programming problem gives us $l_{p(\alpha)}$ and $u_{p(\alpha)}$ as,

$$l_{pm(\alpha)} = Min\, pm(a,s) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \mathbf{(6)}$$

$$\text{such that} \quad l_{AR_i(\alpha)} \leq \mathrm{a} \leq u_{AR_i(\alpha)}, \mathrm{i} = 1,2$$

$$l_{SR(\alpha)} \leq \mathrm{s} \leq u_{SR(\alpha)}$$

and

$$u_{pm(\alpha)} = Maxpm(a,s) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \mathbf{(7)}$$

$$\text{such that} \quad l_{AR_i(\alpha)} \leq \mathrm{a} \leq u_{AR_i(\alpha)}, \mathrm{i} = 1,2$$

$$l_{SR(\alpha)} \leq \mathrm{s} \leq u_{SR(\alpha)}$$

3.5 Construction of shape function from membership function

The lower limit $l_{pm(\alpha)}$ and the upper limit $u_{pm(\alpha)}$ are one- one and onto with respect to α, then the left shape function L(z) = $\left(l_{pm(\alpha)}\right)^{-1}$ and the right shape function R(z) = $\left(u_{pm(\alpha)}\right)^{-1}$ can be obtained, from which the membership function $\mu_{\mathrm{pm}(\widetilde{\mathrm{AR}}_\mathrm{I},\widetilde{\mathrm{SR}})}(z), i = 1,2$

is constructed as

$$\mu_{\mathrm{pm}(\widetilde{\mathrm{AR}}_\mathrm{I},\widetilde{\mathrm{SR}})}(z) = \begin{cases} L(z), & for z_1 \leq z \leq z_2 \\ R(z), & for z_2 \leq z \leq z_3 \\ 0, & otherwise \end{cases} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \mathbf{(8)}$$

Where $z_1 \leq z_2 \leq z_3$ and $\mathrm{L(z_1)} = \mathrm{R(z_3)} = 0$ for the triangular fuzzy number.

3.6 $FM^b/FM/$S reduce to M/M/S

Using the concept of $\alpha -$ cut the $FM^b/FM/$S queue with priority can be reduced to M/M/S in the following form from definition 2.9,

$$\left.\begin{aligned} L_{qP1} &= \frac{\lambda\mu\left(\frac{\lambda_1}{\mu}\right)^s}{(s-1)!(s\mu-\lambda_1)^2} P_0, \\ L_{qP2} &= \frac{\lambda\mu\left(\frac{\lambda_2}{\mu}\right)^s}{(s-1)!(s\mu-\lambda_2)^2} P_0 \end{aligned}\right\} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \mathbf{(9)}$$

$$\left.\begin{aligned} W_{qP1} &= \frac{\mu\left(\frac{\lambda_1}{\mu}\right)^s}{(s-1)!(s\mu-\lambda_1)^2} P_0, \\ W_{qP2} &= \frac{\mu\left(\frac{\lambda_2}{\mu}\right)^s}{(s-1)!(s\mu\ \ \lambda_2)^2} P_0 \end{aligned}\right\} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \mathbf{(10)}$$

and $P_0 = \left[\sum_{n=0}^{s-1} \frac{(\lambda b/\mu)^n}{n!} + \frac{(\lambda b/\mu)^s}{s!} \frac{s\mu}{(s\mu-\lambda)}\right]^{-1}$

Where $\widetilde{\lambda_1}$ and $\widetilde{\lambda_2}$ are the arrival rates of type 1 priority and type 2 priority units respectively and μ is the service rate. Further $\tilde{\lambda} = \widetilde{\lambda_1} + \widetilde{\lambda_2}\cdot$ and 'S' is number of service channel.

The membership functions of the following are derived from the respective parametric program

$\widetilde{L_{qP1}}$ - average queue length of Higher priority

$\widetilde{L_{qP2}}$ - average queue length of lesser priority

$\widetilde{W_{qP1}}$ - average waiting time of units of Higher priority

$\widetilde{W_{qP2}}$ - average waiting time of units of lesser priority.

If the functions $l_{pm(\alpha)}$ and $u_{pm(\alpha)}$ are not one-one and onto with respect to α, the membership functions $\mu_{\text{pm}\,(\tilde{\text{A}},\tilde{\text{S}})}(z)$ in not derived. But we can trace the graph of $\mu_{\text{pm}(\tilde{\text{A}},\tilde{\text{S}})}(z)$ from the $\alpha -$ cuts of $[l_{pm(\alpha)}, u_{pm(\alpha)}]$.

This procedure can be applied to find the membership functions $\mu_{\text{pm}(\widetilde{\text{Arı}},\widetilde{\text{SR}})}(z)$,

$i = 1,2$ for the batch priority queuing model with priority can be obtained.

4. Numerical Example

Expected number of customer and expected waiting time for $FM^b/FM/S$ batch priority queue with two priority types

To calculate the possible number of customers and waiting time in the queue with two priority classes with following assumptions

The arrival rate of the two types of priority queues with the same service rate (ie.) $\mu_1 = \mu_2 = \mu$. Further $\rho_1 = \frac{\widetilde{\lambda_1}}{\widetilde{\mu_1}} = \frac{\widetilde{\lambda_1}}{\widetilde{\mu_2}}, \rho_2 = \frac{\widetilde{\lambda_2}}{\widetilde{\mu_1}} = \frac{\widetilde{\lambda_2}}{\widetilde{\mu_2}}$

$\rho = \rho_1 + \rho_2, \tilde{\lambda} = \widetilde{\lambda_1} + \widetilde{\lambda_2}\cdot$.

The general bulk service rule is used in a multi-server to serve the consumers in batches. The service time of the batches is supposed to rely on the batch size of the continuing service with their required service and follows a general distribution. Since the higher priority queue customer may get service from anyone the service from any ‘S’ services. But the all the ‘S’ service channel will serve uniformly with same service rate $\tilde{\mu}$. (i.e) high priority customer or less priority customer come to service channel and get service from ‘S’ number of different services with same service rate $\tilde{\mu}$.

4.1 Construction of parametric programming problem for Triangular fuzzy numbers

The arrival rate of higher priority is $\widetilde{AR}_1 = [3,4,5], \widetilde{AR}_2 = [6,7,8]$ and $\widetilde{SR} = [14,15,16]$ per hour respectively with batch size b=12 and two service channel. The $\alpha -$ cut of the membership functions $u_{\widetilde{AR}_1}(\alpha), u_{\widetilde{AR}_2}(\alpha)$ and $u_{\widetilde{SR}}(\alpha)$ are $[\alpha + 3, 5 - \alpha], [\alpha + 6, 8 - \alpha]$ and

$[\alpha + 14, 16 - \alpha]$ respectively using definition 2.6. From equation (6) and (7) the parametric programming problem are derived from the respective parametric programs. These differ only in the objective function

$$\left.\begin{array}{ll} \mathrm{L}_{L_{qP1}}(\alpha) \ = min & \left\{\frac{(\mathrm{R1}+\mathrm{R2})\mathrm{T}\left(\frac{\mathrm{R1}}{\mathrm{T}}\right)^{\mathrm{s}}}{(s-1)!(sT-R1)^2}P_0\right\} \\ such\ that & 3+\alpha \le R1 \le 5-\alpha \\ & 6+\alpha \le R2 \le 8-\alpha \\ & 14+\alpha \le T \le 16-\alpha \end{array}\right\} \text{………………}\mathbf{(11)}$$

and

$$\left.\begin{array}{ll} \mathrm{U}_{L_{qP1}}(\alpha) \ = max & \left\{\frac{(\mathrm{R1}+\mathrm{R2})\mathrm{T}\left(\frac{\mathrm{R1}}{\mathrm{T}}\right)^{\mathrm{s}}}{(s-1)!(sT-R1)^2}P_0\right\} \\ such\ that & 3+\alpha \le R1 \le 5-\alpha \\ & 6+\alpha \le R2 \le 8-\alpha \\ & 14+\alpha \le T \le 16-\alpha \end{array}\right\} \text{………………}\mathbf{(12)}$$

Where $0 < \alpha \le 1$.

Also $P_0 = \left[\sum_{n=0}^{s-1} \frac{(\lambda \mathrm{b}/\mu)^n}{n!} + \frac{(\lambda \mathrm{b}/\mu)^s}{s!} \frac{s\mu}{(s\mu-\lambda)}\right]^{-1}$,

If s $= 2$ then ,

$$P_0 = \left[1 + \frac{\lambda \mathrm{b}}{\mu}\left(1 + \frac{\lambda \mathrm{b}}{2\mu - \lambda}\right)\right]^{-1}$$

(ie) $P_0 = 0.01409$ (using λ=11, $\mu = 15, \mathrm{b} = 12$)

$\mathrm{L}_{L_{qP}}\ (\alpha)$ is found when R1 and R2 approaches their lower bounds and T approaches its upper bound.

Consequently, the optimal solutions for (11) is

$$\mathrm{L}_{L_{qP1}}(\alpha) = \frac{2\alpha^3+21\alpha^2+72\alpha+81}{-8\alpha^3+312\alpha^2-3688\alpha+13348}(0.01409) \quad \text{…………}\mathbf{(13)}$$

Also, $U_{L_{q1}(\alpha)}$ is found when R1 and R2 approaches their upper bounds and T approaches its lower bound.

Consequently, the optimal solutions for (12) is

$$\mathrm{U}_{L_{qP_1}}(\alpha) = \frac{-2\alpha^3+33\alpha^2-180\alpha+325}{8\alpha^3+264\alpha^2+2536\alpha+715}\,(0.01409) \qquad \textbf{……………(14)}$$

The membership function

$$\mu_{L_{qP}}\ (z) = \begin{cases} L(z), & [l_{L_{q1}(\alpha)}]_{\alpha=0} \le z \le [l_{L_{q1}(\alpha)}]_{\alpha=1} \\ R(z), & [u_{L_{q1}(\alpha)}]_{\alpha=1} \le z \le [u_{L_{q1}(\alpha)}]_{\alpha=0} \\ 0, & otherwise \end{cases}$$

which is estimated as

$$\mu_{L_{qP_1}}(z) = \begin{cases} L(z), & 0.006068P_0 \le z \le 0.017663P_0 \\ R(z), & 0.017663P_0 \le z \le 0.045416P_0 \\ 0, & otherwise \end{cases} \quad \textbf{….(15)}$$

Similarly the performance functions of $\widetilde{L_{qP2}}$ are derived from the parametric programs.

$$\left.\begin{aligned} \mathrm{L}_{L_{qP}}\ (\alpha)\ &= min \left\{ \frac{(\mathrm{R1+R2})\mathrm{T}\left(\frac{\mathrm{R2}}{\mathrm{T}}\right)^{\mathrm{S}}}{(s-1)!(sT-R2)^2} P_0 \right\} \\ such\ that \quad & 3+\alpha \le R1 \le 5-\alpha \\ & 6+\alpha \le R2 \le 8-\alpha \\ & 14+\alpha \le T \le 16-\alpha \end{aligned}\right\} \textbf{………………(16)}$$

and

$$\left.\begin{aligned} \mathrm{U}_{L_{qP_2}}(\alpha)\ &= max \left\{ \frac{(\mathrm{R1+R2})\mathrm{T}\left(\frac{\mathrm{R2}}{\mathrm{T}}\right)^{\mathrm{S}}}{(s-1)!(sT-R2)^2} P_0 \right\} \\ such\ that \quad & 3+\alpha \le R1 \le 5-\alpha \\ & 6+\alpha \le R2 \le 8-\alpha \\ & 14+\alpha \le T \le 16-\alpha \end{aligned}\right\} \textbf{……………(17)}$$

$\mathrm{L}_{L_{qP}}$ (α) is found when R1 and R2 approaches their lower bounds and T approaches its upper bound.

Consequently, the optimal solutions for (16) is

$$\mathrm{L}_{L_{qP}}\ (\alpha) = \frac{2\alpha^3+33\alpha^2+180\alpha+324}{-8\alpha^3+300\alpha^2-3280\alpha+1038}\,(0.01409) \qquad \textbf{…….….(18)}$$

Also, $U_{L_{q1}(\alpha)}$ is found when R1 and R2 approaches their upper bounds and T approaches its lower bound.

The optimal solutions for (17) is

$\mathrm{U}_{L_{qP_1}}(\alpha) = \frac{-2\alpha^3+45\alpha^2-336\alpha+832}{8\alpha^3+252\alpha^2+2176\alpha+4960}(0.01409)$ **…………(19)**

The membership function

$$\mu_{L_{qP}}(z) = \begin{cases} L(z), & [l_{L_{q1}(\alpha)}]_{\alpha=0} \leq z \leq [l_{L_{q1}(\alpha)}]_{\alpha=1} \\ R(z), & [u_{L_{q1}(\alpha)}]_{\alpha=1} \leq z \leq [u_{L_{q1}(\alpha)}]_{\alpha=0} \\ 0, & otherwise \end{cases}$$

which is estimated as

$$\mu_{L_{qP}}(z) = \begin{cases} L(z), & 0.031202P_0 \leq z \leq 0.072877P_0 \\ R(z), & 0.072877P_0 \leq z \leq 0.167742P_0 \\ 0, & otherwise \end{cases} \quad \textbf{….(20)}$$

Similarly the performance functions of $\widetilde{W_{qP1}}$ and $\widetilde{W_{qP2}}$ are derived from the respective parametric programs.

$$\left.\begin{array}{l} \mathrm{L}_{W_{qP1}}(\alpha) = min\left\{\frac{\mathrm{T}\left(\frac{\mathrm{R1}}{\mathrm{T}}\right)^{\mathrm{s}}}{(s-1)!(sT-R1)^2}P_0\right\} \\ such\ that \quad 3+\alpha \leq R1 \leq 5-\alpha \\ \qquad\qquad 6+\alpha \leq R2 \leq 8-\alpha \\ \qquad\qquad 14+\alpha \leq T \leq 16-\alpha \end{array}\right\} \textbf{………………(21)}$$

and

$$\left.\begin{array}{l} \mathrm{U}_{W_{qP1}}(\alpha) = max\left\{\frac{\mathrm{T}\left(\frac{\mathrm{R1}}{\mathrm{T}}\right)^{\mathrm{s}}}{(s-1)!(sT-R1)^2}P_0\right\} \\ such\ that \quad 3+\alpha \leq R1 \leq 5-\alpha \\ \qquad\qquad 6+\alpha \leq R2 \leq 8-\alpha \\ \qquad\qquad 14+\alpha \leq T \leq 16-\alpha \end{array}\right\} \textbf{………………(22)}$$

Where $0 < \alpha \leq 1$.

$\mathrm{L}_{W_{qP}}\ (\alpha)$ is found when R1 and R2 approaches their lower bounds and T approaches its upper bound.

Consequently, the optimal solutions for (21) is

$$\mathrm{L}_{W_{qP1}}(\alpha) = \frac{\alpha^2+6\alpha+9}{-9\alpha^3+318\alpha^2-3625\alpha+1345}\,(0.01409) \quad \ldots\ldots\ldots\mathbf{(23)}$$

Also, $U_{W_{q1(\alpha)}}$ is found when R1 and R2 approaches their upper bounds and T approaches its lower bound.

Consequently, the optimal solutions for (22) is

$$\mathrm{U}_{W_{qP1}}(\alpha) = \frac{\alpha^2-10\alpha+}{9\alpha^3+264\alpha^2+2461\alpha+7406}\,(0.01409) \quad \ldots\ldots\ldots\ldots\ldots\mathbf{(24)}$$

The membership function

$$\mu_{W_{qP1}}(z) = \begin{cases} L(z), & [l_{L_{q1}(\alpha)}]_{\alpha=0} \leq z \leq [l_{L_{q1}(\alpha)}]_{\alpha=1} \\ R(z), & [u_{L_{q1}(\alpha)}]_{\alpha=1} \leq z \leq [u_{L_{q1}(\alpha)}]_{\alpha=0} \\ 0, & otherwise \end{cases}$$

which is estimated as

$$\mu_{W_{qP1}}(z) = \begin{cases} L(z), & 0.0006688P_0 \leq z \leq 0.0015779P_0 \\ R(z), & 0.0015779P_0 \leq z \leq 0.0033756P_0 \\ 0, & otherwise \end{cases} \ldots\ldots\ldots\mathbf{(25)}$$

In the same way the performance functions of $\widetilde{W_{qP}}$ are derived from the parametric programs.

$$\left.\begin{array}{l} \mathrm{L}_{W_{qP_2}}(\alpha) \ = min \left\{\frac{\mathrm{T}\left(\frac{\mathrm{R2}}{\mathrm{T}}\right)^{\mathrm{s}}}{(s-1)!(sT-R2)^2} P_0\right\} \\ such\ that \qquad 3+\alpha \leq R1 \leq 5-\alpha \\ \qquad\qquad\qquad\ \ 6+\alpha \leq R2 \leq 8-\alpha \\ \qquad\qquad\qquad 14+\alpha \leq T \leq 16-\alpha \end{array}\right\} \text{……………….}\mathbf{(26)}$$

and

$$\left.\begin{array}{l} \mathrm{U}_{W_{qP_2}}(\alpha) \ = max \left\{\frac{\mathrm{T}\left(\frac{\mathrm{R2}}{\mathrm{T}}\right)^{\mathrm{s}}}{(s-1)!(sT-R2)^2} P_0\right\} \\ such\ that \qquad 3+\alpha \leq R1 \leq 5-\alpha \\ \qquad\qquad\qquad\ \ 6+\alpha \leq R2 \leq 8-\alpha \\ \qquad\qquad\qquad 14+\alpha \leq T \leq 16-\alpha \end{array}\right\} \text{……………….}\mathbf{(27)}$$

$\mathrm{L}_{W_{qP_2}}(\alpha)$ is found when R1 and R2 approaches their lower bounds and T approaches its upper bound.

The optimal solutions for (26) is

$$\mathrm{L}_{W_{qP_2}}(\alpha) = \frac{\alpha^2+12\alpha+36}{-9\alpha^3+300\alpha^2-3172\alpha+1081}\,(0.01409) \quad \text{…………….}\mathbf{(28)}$$

Also, $U_{W_{q1(\alpha)}}$ is found when R1 and R2 approaches their upper bounds and T approaches its lower bound.

And the optimal solutions for (27) is

$$\mathrm{U}_{W_{qP_1}}(\alpha) = \frac{\alpha^2-16\alpha+64}{9\alpha^3+246\alpha^2+2080\alpha+560}\,(0.01409) \qquad \text{…...……….}\mathbf{(29)}$$

The membership function

$$\mu_{W_{qP}}\ (z) = \begin{cases} L(z), & [l_{L_{q1}(\alpha)}]_{\alpha=0} \leq z \leq\ [l_{L_{q1}(\alpha)}]_{\alpha=1} \\ R(z), & [u_{L_{q1}(\alpha)}]_{\alpha=1} \leq z \leq\ [u_{L_{q1}(\alpha)}]_{\alpha=0} \\ \qquad 0, & \qquad\qquad otherwise \end{cases}$$

which is estimated as

$$\mu_{W_{qP_2}}(z)=\begin{cases} L(z), & 0.0033284P_0 \le z \le 0.0061752P_0 \\ R(z), & 0.0061752P_0 \le z \le 0.0114286P_0 \\ 0, & otherwise \end{cases} \quad \textbf{....(30)}$$

TABLE -1 The α-cuts of the performance measurelength of the queue for priority 1 and priority 2 atdifferent α values

LENGTH OF THE QUEUE

For**Priority 1** Lq_ L P1 is LOWER VALUEof P1, Lq_UP1 is UPPER VALUE of P1 , &
Priority 2 Lq_ L P2 is LOWER VALUE of P2, Lq_UP2 is UPPER VALUE of P2 .

α	Lq_L P1	Lq_U P1	Lq_L P2	Lq_U P2
0	0.0000855027	0.0006399176	0.0004396341	0.002363484
0.1	0.0000959555	0.0005842022	0.0004795170	0.002172875
0.2	0.0001074441	0.0005330306	0.0005227164	0.001998239
0.3	0.0001200562	0.0004860327	0.0005695079	0.001838089
0.4	0.0001338869	0.0004428704	0.0006201926	0.001691098
0.5	0.0001490394	0.0004032352	0.0006750992	0.001556078
0.6	0.0001656251	0.0003668449	0.0007345877	0.001431966
0.7	0.0001837651	0.0003334413	0.0007990521	0.001317805
0.8	0.0002035905	0.0003027876	0.0008689246	0.001212735
0.9	0.0002252439	0.0002746669	0.0009446799	0.001115980
1	0.0002488800	0.0002488800	0.0010268402	0.001026840

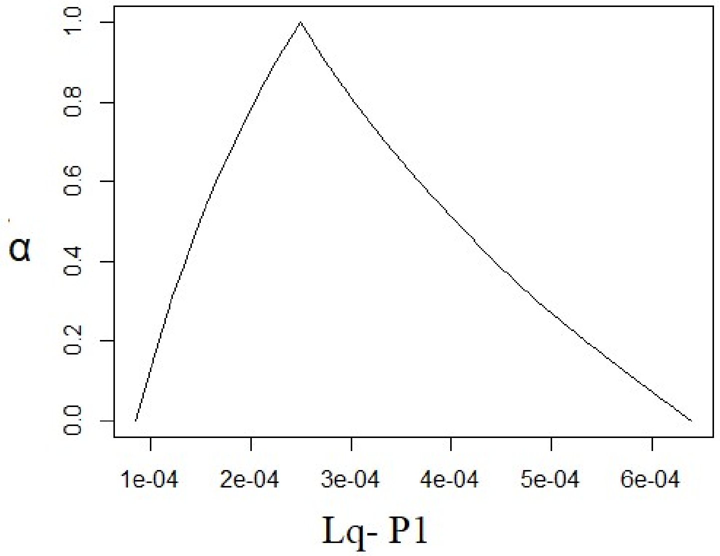

Fig. 1 The membership function of the Expected queue length Lq of priority 1

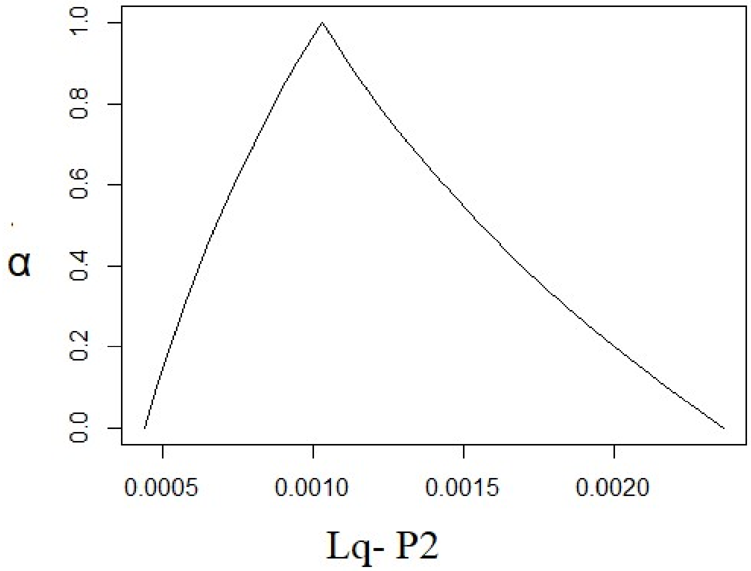

Fig. 2 The membership function of expected queue length Lq of priority 2

TABLE -2 Theα-cuts of the performance measure waiting time in the queue for priority 1 and priority 2 at different α values

WAITING TIME IN THE QUEUE

for **Priority 1** Wq_ L P1 is LOWER VALUE of P1, Wq_UP1 is UPPER VALUE of P1 , &
Priority 2 Wq_ L P2 is LOWER VALUE of P2, Wq_UP2 is UPPER VALUE of P2 .

α	Wq_L P1	Wq_UP1	Wq_L P2	Wq U P2
0	0.0000094240	0.0000475628	0.00004689719	0.0001610286
0.1	0.0000103386	0.0000441949	0.00004992376	0.0001513401
0.2	0.0000113213	0.0000410470	0.00005313367	0.0001422582
0.3	0.0000123764	0.0000381044	0.00005653859	0.0001337408
0.4	0.0000135086	0.0000353536	0.00006015102	0.0001257488
0.5	0.0000147229	0.0000327820	0.00006398436	0.0001182466
0.6	0.0000160247	0.0000303781	0.00006805302	0.0001112010
0.7	0.0000174195	0.0000281309	0.00007237247	0.0001045815
0.8	0.0000189136	0.0000260304	0.00007695932	0.00009835994
0.9	0.0000205135	0.0000240672	0.00008183148	0.00009251022
1	0.0000222262	0.0000222327	0.00008700819	0.00008700819

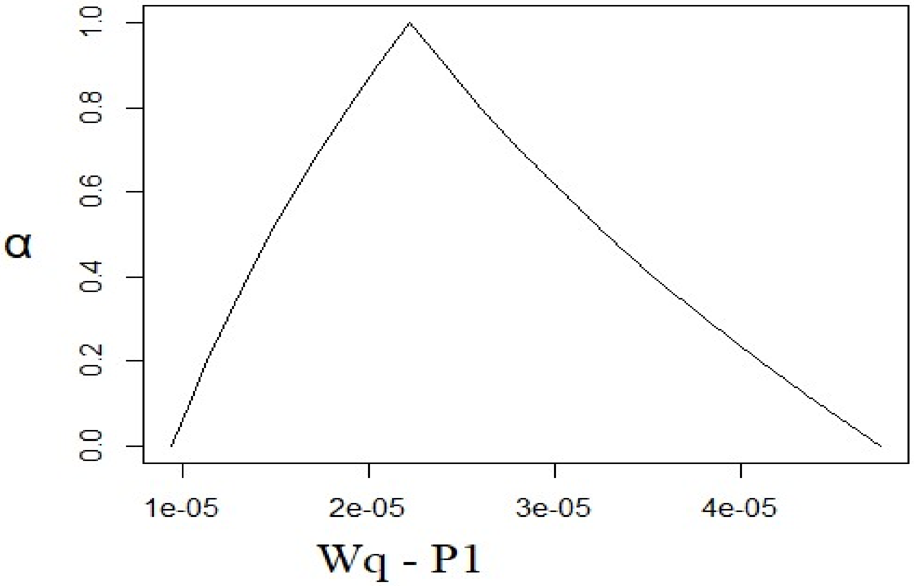

Fig. 3 The membership function of the expected waiting time Wq of priority 1

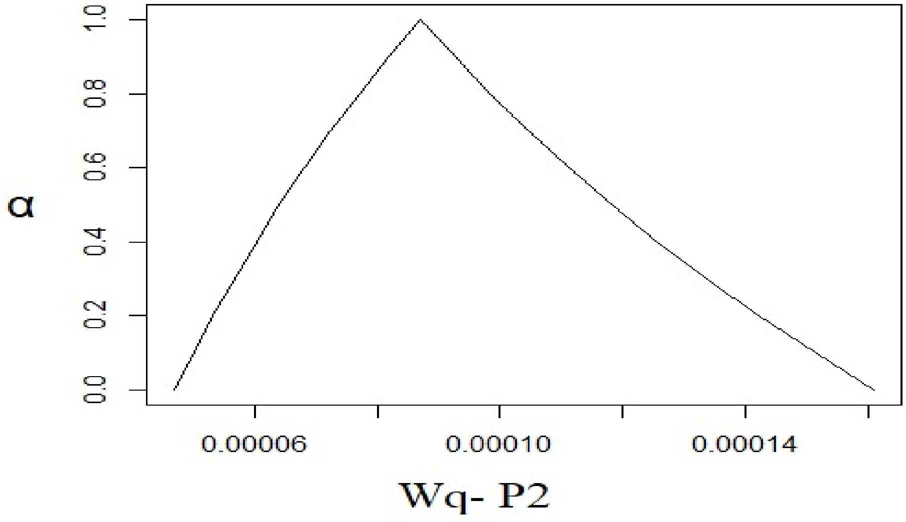

Fig. 4 The membership function of the expected waiting time Wq of priority 2

4.2 Construction of parametric programming problem for Trapezoidal fuzzy numbers

The arrival rate of higher priority is $\widetilde{AR}_1 = [2,3,4,5], \widetilde{AR}_2 = [6,7,8,9]$ and $\widetilde{SR} = [13,14,15,16]$ per hour respectively with batch size b=12 and two service channel. The $\alpha -$ cut of the membership functions $u_{\widetilde{AR}_1}(\alpha), u_{\widetilde{AR}_2}(\alpha)$ and $u_{\widetilde{SR}}(\alpha)$ are $[\alpha + 2,5 - \alpha], [\alpha + 6,98 - \alpha]$ and

$[\alpha + 13, 16 - \alpha]$ respectively using definition 2.8.

Construction of shape function from membership function of trapezoidal fuzzy number

The lower limit $l_{pm(\alpha)}$ and the upper limit $u_{pm(\alpha)}$ are one- one and onto with respect to α, then the left shape function L(z) $= \left(l_{pm(\alpha)}\right)^{-1}$ and the right shape function R(z) $= \left(u_{pm(\alpha)}\right)^{-1}$ can be obtained, from which the membership function $\mu_{\mathrm{pm}(\widetilde{\mathrm{AR}}1,\widetilde{\mathrm{SR}})}(z), i = 1,2$

is constructed as

$$\mu_{\mathrm{pm}(\widetilde{\mathrm{AR}}1,\widetilde{\mathrm{SR}})}(z) = \begin{cases} L(z), & for z_1 \le z \le z_2 \\ 1, & for z_2 \le z \le z_3 \\ R(z), & for z_3 \le z \le z_4 \\ 0, & otherwise \end{cases} \text{.............}\textbf{(31)}$$

Where $z_1 \le z_2 \le z_3 \le z_4$ and $\mathrm{L(z_1)} = \mathrm{R(z_4)} = 0$ for the trapezoidal fuzzy number.

From equation (8) and (9) the parametric programming problem are derived from the respective parametric programs.

$$\left.\begin{array}{l} \mathrm{L}_{\square\square\square 1}(\square) = \square\square\square \left\{ \frac{(\mathrm{R1}+\mathrm{R2})\mathrm{T}\left(\frac{\mathrm{R1}}{\mathrm{T}}\right)^{\mathrm{s}}}{(\square-1)!(\square\square-\square 1)^{2}} \square_{0} \right\} \\ \square\square\square h\ \square h\square\square \qquad 2+\square \le \square 1 \le 5-\square \\ \qquad\qquad\qquad 6+\square \le \square 2 \le 9-\square \\ \qquad\qquad\qquad 13+\square \le \square \le 16-\square \end{array}\right\} \text{..................}\textbf{(32)}$$

and

$$\left.\begin{array}{l} U_{\square\square\square 1}(\square) = \square\square\square \left\{ \frac{(R1+R2)T\left(\frac{R1}{T}\right)^{s}}{(\square-1)!(\square\square-\square 1)^{2}} \square_{0} \right\} \\ \square\square\square h\ \square h\square\square \qquad 2+\square \le \square 1 \le 5-\square \\ \qquad\qquad\qquad 6+\square \le \square 2 \le 9-\square \\ \qquad\qquad\qquad 13+\square \le \square \le 16-\square \end{array}\right\} \cdots\cdots\cdots\cdots\cdots\cdots \mathbf{(33)}$$

$$\left.\begin{array}{l} L_{\square\square\square 2}(\square) = \square\square\square \left\{ \frac{(R1+R2)T\left(\frac{R2}{T}\right)^{s}}{(\square-1)!(\square\square-\square 2)^{2}} \square_{0} \right\} \\ \square\square\square h\square h\square\square \qquad 2+\square \le \square 1 \le 5-\square \\ \qquad\qquad\qquad 6+\square \le \square 2 \le 9-\square \\ \qquad\qquad\qquad 13+\square \le \square \le 16-\square \end{array}\right\} \cdots\cdots\cdots\cdots\cdots\cdots \mathbf{(34)}$$

and

$$\left.\begin{array}{l} U_{\square\square\square 2}(\square) = \square\square\square \left\{ \frac{(R1+R2)T\left(\frac{R2}{T}\right)^{s}}{(\square-1)!(\square\square-\square 2)^{2}} \square_{0} \right\} \\ \square\square\square h\ \square h\square\square \qquad 2+\square \le \square 1 \le 5-\square \\ \qquad\qquad\qquad 6+\square \le \square 2 \le 9-\square \\ \qquad\qquad\qquad 13+\square \le \square \le 16-\square \end{array}\right\} \cdots\cdots\cdots\cdots\cdots\cdots \mathbf{(35)}$$

$$\left.\begin{array}{l} L_{\square\square\square 1}(\square) = \square\square\square \left\{ \frac{T\left(\frac{R1}{T}\right)^{s}}{(\square-1)!(\square\square-\square 1)^{2}} \square_{0} \right\} \\ \square\square\square h\square h\square\square \qquad 2+\square \le \square 1 \le 5-\square \\ \qquad\qquad\qquad 6+\square \le \square 2 \le 9-\square \\ \qquad\qquad\qquad 13+\square \le \square \le 16-\square \end{array}\right\} \cdots\cdots\cdots\cdots\cdots\cdots \mathbf{(36)}$$

and

$$\left.\begin{array}{l} U_{\square\square\square 1}(\square) = \square\square\square \left\{ \frac{T\left(\frac{R1}{T}\right)^{s}}{(\square-1)!(\square\square-\square 1)^{2}} \square_{0} \right\} \\ \square\square\square h\ \square h\square\square \qquad 2+\square \le \square 1 \le 5-\square \\ \qquad\qquad\qquad 6+\square \le \square 2 \le 9-\square \\ \qquad\qquad\qquad 13+\square \le \square \le 16-\square \end{array}\right\} \cdots\cdots\cdots\cdots\cdots\cdots \mathbf{(37)}$$

$$\left.\begin{array}{l} L_{\square\square\square 2}(\square) = \square\square\square \left\{ \frac{T\left(\frac{R2}{T}\right)^{s}}{(\square-1)!(\square\square-\square 2)^{2}} \square_{0} \right\} \\ \square\square\square h\ \square h\square\square \qquad 2+\square \le \square 1 \le 5-\square \\ \qquad\qquad\qquad 6+\square \le \square 2 \le 9-\square \\ \qquad\qquad\qquad 13+\square \le \square \le 16-\square \end{array}\right\} \cdots\cdots\cdots\cdots\cdots\cdots \mathbf{(38)}$$

and

$$\left.\begin{array}{l} \mathrm{U}_{\square\square\square 2}(\square) = m\square\square \left\{\frac{\mathrm{T}\left(\frac{\mathrm{R2}}{\mathrm{T}}\right)^{\mathrm{s}}}{(\square-1)!(\square\square-\square 2)^{2}}\square_{0}\right\} \\ \square\square\square h\ \square h\square\square \qquad \begin{array}{r} 2+\square \leq \square 1 \leq 5-\square \\ 6+\square \leq \square 2 \leq 9-\square \\ 13+\square \leq \square \leq 16-\square \end{array} \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\mathbf{(39)}$$

Where $0 < \square \leq 1$.

$\mathrm{L}_{\square\square\square 1}(\square), \mathrm{L}_{\square\square\square 2}(\square), \mathrm{L}_{\square\square\square 1}(\square)$ and $\mathrm{L}_{\square\square\square 2}(\square)$ are found when R1 and R2 approaches their lower bounds and T approaches its upper bound.

Then, $\mathrm{U}_{\square\square\square 1}(\square), \mathrm{U}_{\square\square\square 2}(\square), \mathrm{U}_{\square\square\square 1}(\square)$ and $\mathrm{U}_{\square\square\square 2}(\square)$ are found when R1 and R2 approaches their upper bounds and T approaches its lower bound.

Also $\square_{0} = \left[\sum_{\square=0}^{\square-1}\frac{(\lambda \mathrm{b}/\mu)^{\square}}{\square!} + \frac{(\lambda \mathrm{b}/\mu)^{\square}}{\square!}\frac{\square\square}{(\square\square-\lambda)}\right]^{-1}$,

If $\mathrm{s} = 2$ then,

$$\square_{0} = \left[1 + \frac{\lambda \mathrm{b}}{\square}\left(1 + \frac{\lambda \mathrm{b}}{2\square - \lambda}\right)\right]^{-1}$$

(ie) $\square_{0} = 0.01301$ (using λ=11, $\mu = 14.5, \mathrm{b} = 12$)

Consequently, the optimal solutions for (32), (33) are

$$\mathrm{L}_{\square\square\square 1}(\square) = \frac{2\square^{3}+16\square^{2}+40\square+32}{-9\square^{3}+324\square^{2}-3780\square+14400}(0.01301) \quad \ldots\ldots\ldots..\mathbf{(40)}$$

$$\mathrm{U}_{\square\square\square 1}(\square) = \frac{-2\square^{3}+34\square^{2}-190\square+350}{9\square^{3}+243\square^{2}+2079\square+5733}(0.01301) \quad \ldots\ldots\ldots\ldots\mathbf{(41)}$$

And optimal solutions for (34), (35) are

$$\mathrm{L}_{\square\square\square 2}(\square) = \frac{2\square^{3}+32\square^{2}+168\square+288}{-9\square^{3}+300\square^{2}-3172\square+10816}(0.01301) \quad \ldots\ldots\ldots..\mathbf{(42)}$$

$$U_{\square_{\square\square 2}}(\square) = \frac{-2\square^{3}+50\square^{2}-414\square+1134}{9\square^{3}+219\square^{2}+1615\square+3757}(0.01301) \quad \text{……….}\mathbf{(43)}$$

Then optimal solutions for (36), (37) are

$$L_{\square_{\square\square 1}}(\square) = \frac{\square^{2}+4\square+4}{-9\square^{3}+324\square^{2}-3780\square+14400}(0.01301) \quad \text{…………}\mathbf{(44)}$$

$$U_{\square_{\square\square 1}}(\square) = \frac{\square^{2}-10\square+25}{9\square^{3}+243\square^{2}+2079\square+5733}(0.01301) \quad \text{…………}\mathbf{(45)}$$

Also, optimal solutions for (38), (39) are

$$L_{\square_{\square\square 2}}(\square) = \frac{\square^{2}+12\square+36}{-9\square^{3}+300\square^{2}-3172\square+10816}(0.01301)\text{……..……}\mathbf{(46)}$$

$$U_{\square_{\square\square 2}}(\square) = \frac{\square^{2}-18\square+81}{9\square^{3}+219\square^{2}+1615\square+3757}(0.01301) \quad \text{…..………}\mathbf{(47)}$$

The membership function

$$\square_{\square_{\square\square 1}}(\square) = \begin{cases} \square(\square), & [\square_{\square_{\square 1}(\square)}]_{\square=0} \leq \square \leq [\square_{\square_{\square 1}(\square)}]_{\square=1} \\ \square(\square), & [\square_{\square_{\square 1}(\square)}]_{\square=1} \leq \square \leq [\square_{\square_{\square 1}(\square)}]_{\square=0} \\ 0, & \square\square h\square\square\square\square\square\square\square \end{cases}$$

which is estimated as

$$\square_{\square_{\square\square 1}}(\square) = \begin{cases} \square(\square), & 0.0022222\square_{0} \leq \square \leq 0.0082304\square_{0} \\ \square(\square), & 0.0238095\square_{0} \leq \square \leq 0.0434027\square_{0} \\ 0, & \square\square h\square\square\square\square\square\square\square \end{cases} \text{…..}\mathbf{(48)}$$

$$\square_{\square_{\square\square 2}}(\square) = \begin{cases} \square(\square), & 0.026663\square_{0} \leq \square \leq 0.061752\square_{0} \\ \square(\square), & 0.137142\square_{0} \leq \square \leq 0.301837\square_{0} \\ 0, & \square\square h\square\square\square\square\square\square\square \end{cases} \text{…….}\mathbf{(49)}$$

$$\square_{\square_{\square\square 1}}(\square) = \begin{cases} \square(\square), & 0.0002777\square_{0} \leq \square \leq 0.0008234\square_{0} \\ \square(\square), & 0.00198413\square_{0} \leq \square \leq 0.0043607\square_{0} \\ 0, & \square\square h\square\square\square\square\square\square\square \end{cases} \text{…….}\mathbf{(50)} \quad \square_{\square_{\square\square 2}}(\square) =$$

$$\begin{cases} \square(\square), & 0.003328\square_0 \leq \square \leq 0.0061752\square_0 \\ \square(\square), & 0.0114285\square_0 \leq \square \leq 0.0215598\square_0 \\ 0, & \square\square h\square\square\square\square\square\square \end{cases} \quad \text{.......}(51)$$

TABLE -3 The α-cuts of the performance measure length of the queue for priority 1 and priority 2 at different α values (Trapezoidal fuzzy number)

Trapezoidal fuzzy number LENGTH OF THE QUEUE

for **Priority 1** Lq_ L P1 is LOWER VALUE of P1, Lq_UP1 is UPPER VALUE of P1 , &
Priority 2 Lq_ L P2 is LOWER VALUE of P2, Lq_UP2 is UPPER VALUE of P2 .

α	Lq_L P1	Lq_U P1	Lq_L P2	Lq_U P2
0	0.00002891111	0.0007942613	0.0003464201	0.003926894
0.1	0.00003354438	0.0007253006	0.0003779962	0.003627212
0.2	0.00003873024	0.0006619379	0.0004121122	0.003351061
0.3	0.00004451904	0.0006037233	0.0004489622	0.003096442
0.4	0.00005096510	0.0005502450	0.0004887558	0.002861545
0.5	0.00005812707	0.0005011259	0.0005317196	0.002644730
0.6	0.00006606826	0.0004560201	0.0005780981	0.002444507
0.7	0.00007485707	0.0004146104	0.0006281560	0.002259523
0.8	0.00008456737	0.0003766056	0.0006821797	0.002088546
0.9	0.00009527899	0.0003417381	0.0007404790	0.001930456
1	0.0001070782	0.0003097619	0.0008033900	0.001784229

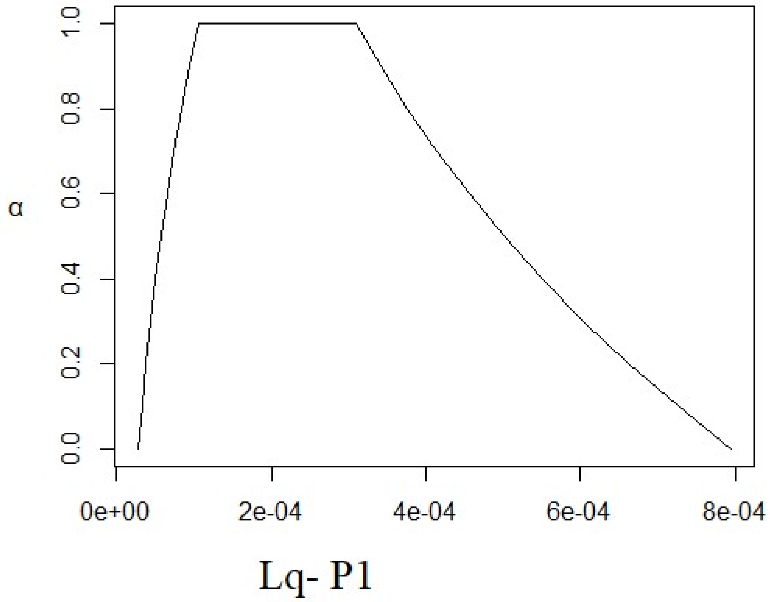

Fig. 5 The membership function of the Trapezoidal Expectedqueue length Lq of priority 1

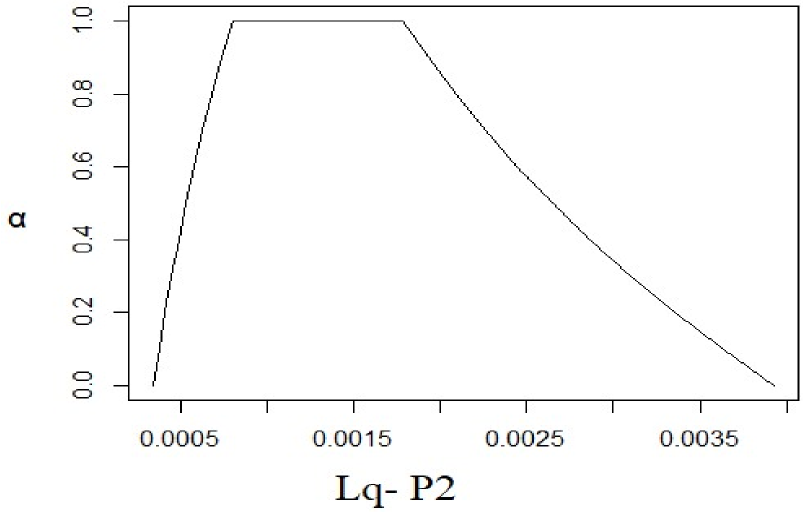

Fig. 6 The membership function of the Trapezoidal Expectedqueue length Lq of priority 2

TABLE -4 The α-cuts of the performance measure waiting time in the queue for priority 1 and priority 2 at different α values(Trapezoidal fuzzy number)

Trapezoidal fuzzy number WAITING TIME IN THE QUEUE

for **Priority 1** Wq_ L P1 is LOWER VALUE of P1, Wq_UP1 is UPPER VALUE of P1 , &
Priority 2 Wq_ L P2 is LOWER VALUE of P2, Wq_UP2 is UPPER VALUE of P2 .

α	Wq_L P1	Wq_UP1	Wq_L P2	Wq UP2
0	0.00000361389	0.00005673295	0.00004330251	0.0002804924
0.1	0.00000409078	0.00005255801	0.00004609710	0.0002628414
0.2	0.00000461074	0.00004867190	0.00004906097	0.0002464015
0.3	0.00000517663	0.00004505398	0.00005220490	0.0002310777
0.4	0.00000579149	0.00004168523	0.00005554044	0.0002167837
0.5	0.00000645856	0.00003854815	0.00005907995	0.0002034407
0.6	0.00000718133	0.00003562657	0.00006283675	0.0001909771
0.7	0.00000796352	0.00003290559	0.00006682511	0.0001793272
0.8	0.00000880910	0.00003037142	0.00007106038	0.0001684312
0.9	0.00000972235	0.00002801132	0.00007555908	0.0001582341
1	0.00001070782	0.00002581349	0.00008033900	0.0001486857

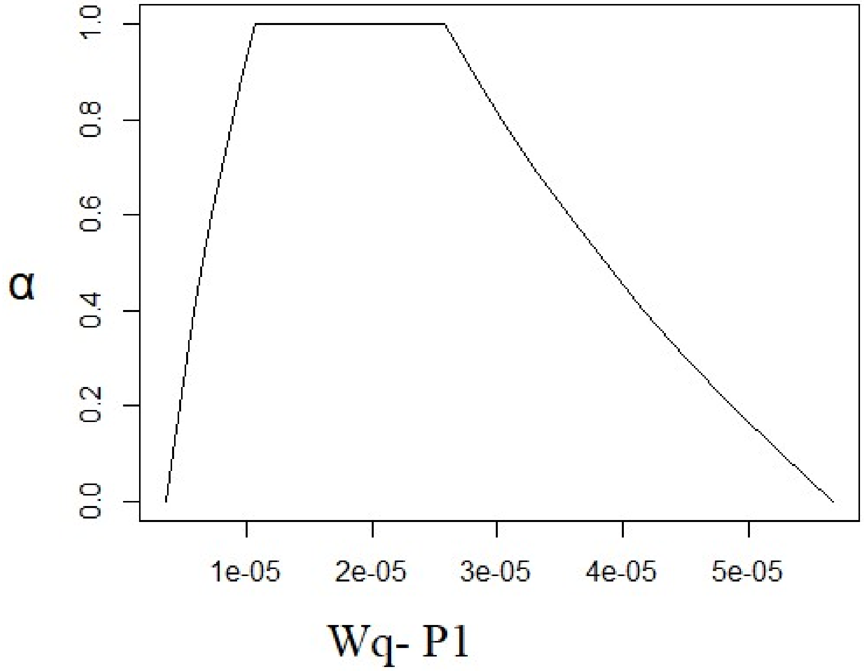

Fig. 7 The membership function of the Trapezoidal expected waiting time Wq of priority 1

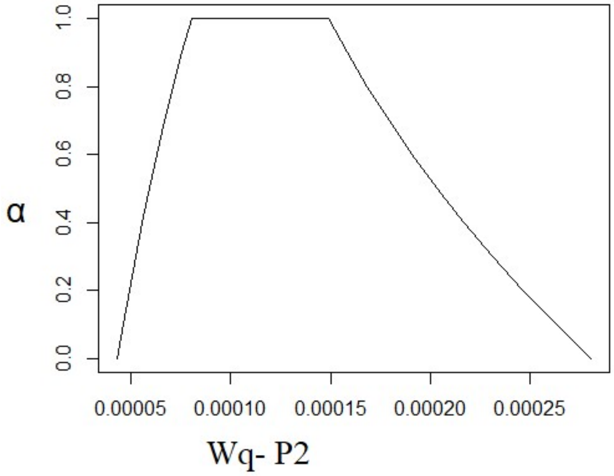

Fig. 8 The membership function of the Trapezoidal expected waiting time Wq of priority 2

We perform α-cuts of batch arrival size, arrival rate, service rate, and fuzzy estimated number of customers in priority queues with the aid of R programming at various levels: 0, 0.1, 0.2....1. **Table 1** displays clear intervals for the fuzzy expected number of customers in priority queues at various levels of probabilistic for customers.

Similarly fuzzy expected waiting time in queue at various levels of probabilistic for priority customers also derived in Table 1. Here we enumerate 22 values of α = 0.1, 0.2, 0.3..., 1.0. **Fig.1** depicts the rough shape of Lq P1 and Lq_P2 constructed from these 22,α values. The rough shape turns out rather fine and looks like a continuous function. Other performance measures are depicted by the remaining figuresFig-3 to Fig-8. The α-cut represent the possibility that these performance measure will lie in the associated range.

Specially, α = 0 the range, the performance measures could appear and for

α = 1 the range, the performance measures are likely to be. For example table 1, while these four performance measures are fuzzy, the most likely value of the expected queue length lower value of Lq_P1 falls between 0.000855027 and 0.000248800 and its value is impossible to fall outside the range of 0.0008549812 and 0.000248800; it is definitely possible that from **table 2,** the expected waiting time in the queuelower value of Wq_P1 falls between 0.0000094240hours and 0.0000222262hours, and it will never fall below 0.000009424048or exceed 0.0.000022232611.

For trapezoidal fuzzy number, from**table 3 and table 4**, the rangebetweenα = 0 andα = 1, are satisfied for all performance measures. From table3, the most likely value of the expected queue length lower value of Lq_LP1 falls between 0.00002891111 and 0.0001070782 and its value is impossible to fall outside the range of 0.000028910822 and 0.0001070782; also for upper value of Lq_UP1 falls between 0.0007942613 and 0.0003097619 and its value is impossible to fall outside the range of 0.00030976159 and 0.00056466912.

It is possible that from **table 4,** the expected waiting time in the queue lower value of Wq_LP1 falls between 0.00000361389hours and 0.00001070782 hours, and it will never fall below 0.000003612877or exceed 0.000010712434. In the same waythe upper value of Wq_UP1 falls between 0.00005673295 and 0.00002581349 and its valuewill never fall below 0.000056732707and fall above 0.0000258135313.

The above information will be very useful for designing a queuing system.

5.CONCLUSION

There are many more uses for bulk arrival queuing models in operations and service processes to prioritize and measure system performance. It will be used in situations such as senior citizen ticket purchase, special admission at airports for VIPs and business class passengers or crew members, senior officer in lifts, hospital emergencies, etc. In this work, where the batch arrival rate with size ‘b’, service rate are fuzzy and the parametric programming problem approach has been used to calculate the membership function of the system performance measure with priority. Based on Zadeh's extension principle, this model can be used to transform a family of bulk arrival crisp priority queues from a bulk arrival priority fuzzy queue. We employed the trapezoidal $FM^{[b]}$/FM/S model.

We were able to calculate performance measure with this model as an alternative to applying a crisp value since the membership function retains the fuzziness of the input data, enabling a more realistic representation of the fuzzy system. Using a numerical example, we were able illustrate how more useful this technique is in real-world situations. It is evident that in addition to triangular and trapezoidal fuzzy numbers, the suggested method may also be applied to other fuzzy numbers, such as pentagonal, octagonal, and decagonal fuzzy numbers.

References

1) Chanas.S, Nowakowshi.M, Single valuesimulationoffuzzyvariable, Fuzzysetsandsystem,21(1988)43-57.
2) Chandhary.M.L, Templeton.J.G.C,AFirstcourseinBulkQueues,JohnWileyandSons,NewYork1983.
3) Devaraj.J, Jayalakshmi.D, A Fuzzy approach to priority queues, International Journal of Fuzzy Mathematics and Systems. ISSN 2248-9940, vol 2, 4(2012) p.479-488.
4) Jau-Chuan Ke, Hsin-I Huang, Chuen-Horng Lin, A batch-arrival queue with multiple servers and fuzzy parameters: Parametric programming approach, Mathematical and Computational Applications, Vol. 11, 3(3006), 181-191
5) Kalpana.B,Anusheela.N, Analysis of FM/FM/I fuzzy priority queues based on new approach of ranking fuzzy numbers using centroid of centroids, International Journal of Pure and Applied Mathematics, 119(7)(2018), 457-465.
6) Kalpana.B, Evaluation of performance measures of fuzzy queues with preemptive priority using different fuzzy numbers, Advances and Applications in Mathematical Sciences, 20(11) (2021) 2975-2985.
7) Kao.C, Li.C.C, Chen.S.P, Parametric Programming to the analysis of fuzzy queue, Fuzzy sets and system, 107(1999) 93-100.
8) Kaufmann.A, Introduction to the Theory of Fuzzy subsets, Vol-I, Academic Press, New York, 1975.
9) Li.R.J, Lee.E.S, Analysis of fuzzy queues, Computers and Mathematics with Applications 17 (1989) 1143- 1147.

10) Nagoor Gani.A, Ashok kumar.V, A Bulk arrival Model with Fuzzy Parameters and Fuzzy varying batch sizes, International journal of algorithms,

Vol 2, 3(2009).

11) Narayanamoorthy.S, Ramya.L, Multi Server Fuzzy Queuing Model Using DSW Algorithm With Hexagonal Fuzzy Number, International Journal of Pure and Applied Mathematics,117(11) (2017), 385-392.

12) Negi.D.S, Lee.E.S, Analysis and simulation of fuzzy queues, Fuzzy Sets and Systems 46 (1992) 321-330.

13) Sharma.R, Sharma.G.C, A queuing model with the bulk arrival rate depending upon the nature of service, Appl. Math. Model 20 (1996),470-475.

14) Zadeh.L.A, Fuzzy sets as a basis for a theory of possibility, Fuzzy sets and system, 1(1978) 3-28.